B. Sc. B.Ed. SEMESTER I EXAMINATION 2019

Subject: Physics

CC 1 (Mathematical Physics - I)

FULL MARKS: 50 TIME ALLOWED: 2 HOURS

Answer any **Ten** (10) questions

1. (i) Prove that if $y^3 - 3ax^2 + x^3 = 0$ then $\frac{d^2y}{dx^2} + \frac{2a^2x^2}{y^5} = 0$ (2 marks)

(ii) Find $\frac{dz}{dt}$ using chain rule if $z = xy^2 + x^2y$, $x = at^2$, $y = 2at$ (3 marks)

2.(i) State Taylor's theorem in two variables. (2 marks)

(ii) If $\vec{F} = (5xy - 6x^2)\hat{i} + (2y - 4x)\hat{j}$, then find the value of $\int \vec{F} \cdot d\vec{r}$ along a curve $y = x^3$ in the xy plane from $(1, 1)$ to $(2, 8)$. (3 marks)

3. State and prove Gauss' Divergence theorem. (5 marks)

4. (i) Explain the meaning of $\boldsymbol{\nabla} |\vec{r}|$ where $\vec{r}$ is the position vector. (2 marks)

(ii) Show that $\boldsymbol{\nabla}^2(\frac{x}{r^3}) = 0$. (3 marks)

5. (i) For three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ prove that $\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$ (3 marks)

(ii) Compute the norm of $Y = \begin{pmatrix} 3 \\ -1 \\ 0 \\ -1 \end{pmatrix}$ (2 marks)

6. (i) If $\Phi(x, y, z) = xy^2z$ and $\vec{A} = xzi - xy^2j + yz^2k$ then find $\frac{\partial^3}{\partial x^2 \partial z}(\Phi\vec{A})$ at $(2, 0, 1)$ (2 marks)

(ii) Use Lagrange multipliers to find the minimum and maximum of the function $f(x, y) = (3x + y)$ subject to the constraint $x^2 + y^2 = 10$ (3 marks)

7. (i) Define Gradient. (1 marks)

(ii) What does it mean physically? (1 marks)

(iii) If $\vec{R} = xi + yj + zk$ then find gradient of $\frac{1}{R}$. (3 marks)

8. Prove that

(i) $\boldsymbol{\nabla} \times \boldsymbol{\nabla}\phi = 0$ (2 marks)

(ii) $\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} \times \vec{A} = 0$ (3 marks)

9. Consider an operator A represented by the following matrix:

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

(i) Verify whether the operator is Hermitian. (1 marks)

(ii) Verify whether the operator is Unitary. (1 marks)

(iii What are the eigen values and eigen vectors of the operator? (3 marks)

10. Consider the matrix $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

(i) Verify whether A is orthogonal. (2 marks)

(ii) Find the eigenvalues of $X = \begin{pmatrix} -1 & 2 & 2 \\ 2 & 2 & 2 \\ -3 & -6 & -6 \end{pmatrix}$ (3 marks)

11. (i) Justify with reason whether the following equation is homogeneous $\frac{dy}{dx} = \frac{2x-y}{x-3y}$ (2 marks)

(ii) Solve the following:
$(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0,\ y(0) = 1$ (3 marks)

12. Solve the following:
$m\frac{d^2x}{dy^2} + \alpha\frac{dy}{dx} + \beta x = k\cos(\omega x),\ y(0) = 0$ (5 marks)